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A model of producer incentives for livestock disease management

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Abstract We examine the management of livestock diseases from the producers' perspective, incorporating information and incentive asymmetries between producers and regulators. Using a stochastic dynamic model, we examine responses to different policy options including indemnity payments, subsidies to report at-risk animals, monitoring, and regulatory approaches to decreasing infection risks when perverse incentives and multiple policies interact. This conceptual analysis illustrates the importance of designing efficient combinations of regulatory and incentive-based policies.

Keywords Livestock disease · Asymmetric information · Reporting · Indemnities · Risk management

1 Introduction

Governments are under pressure to manage the threat of livestock diseases because of public health concerns and the negative impacts on livestock producers. Traditional policies for addressing livestock diseases such as foot and mouth disease and bovine spongiform encephalopathy (mad cow disease) include testing and monitoring activities, conducted by the government, and regulations imposed on livestock producers and processors. Such policies might have limited success, however, if producers do not cooperate with the government. Payments for reporting sick animals, indemnity payments for livestock destroyed for disease control, and other incentive-based policies could encourage producers to aid in disease detection. By creating a suboptimal mix of incentives, however, regulators could fail to reduce, and even exacerbate disease outbreaks. If indemnities are too high, producers could find it beneficial to submit low-value animals for testing or increase the probability of disease outbreaks. With insufficient indemnity payments, producers may slaughter too many animals to avoid future losses; similarly, regulatory policies that ban the use of sick animals may promote early slaughter to avoid detection.

Designing policies to address animal diseases requires understanding the incentives faced by livestock owners. In this article, we develop a model to examine livestock disease management when both the government and producers can affect disease risks. Economic studies of livestock diseases have focused on the effects of health concerns on prices (Piggott and Marsh 2004; Lloyd et al. 2001) and on estimating potential economic impacts (Matthews and Buzby 2001; Matthews and Perry 2003). Studies of livestock owner behavior and livestock populations focus chiefly on explaining cyclical patterns in livestocks (e.g. Aadland 2004). Bicknell et al. (1999) examine livestock owners' incentives to control bovine tuberculosis. In their model, producers select marketing levels, private testing, and eradication of wild animal vectors based upon prices, biological parameters, indemnity payments, the cost and efficacy of testing, and government monitoring of slaughterhouse activity. When government monitoring is 100% effective and producers have no private information about disease infection, government policies are shown to reduce aggregated disease outbreaks, as well as private incentives to control disease.

Using a stochastic dynamic model, we examine the incentives of livestock producers to take private actions

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that can increase or decrease the potential for disease detection. We incorporate the asymmetry of information between producers and government regulators. Producers maximize expected economic benefits from livestock sales and government incentive payments in pre and post disease-detection scenarios. Livestock producers make decisions over harvest, reporting, and other activities that aid government monitoring efforts. They consider expected prices before and after disease detection, government incentive payments, and subjective probabilities of disease detection that can be influenced by federal monitoring and by producers themselves via reporting and other activities such as disposal methods of sick animals. We use this model to examine producers' responses to a range of policy options including indemnity payments for destroyed livestock, subsidies to voluntarily aid in disease detection, government monitoring, and regulations that raise the cost of maintaining livestock. We also consider measures that reduce demand losses upon disease detection, such as improving animal tracking and identification systems.

The analysis characterizes the complex incentives produced by multiple related policies. Increased monitoring by the government, regulations to reduce disease transmission, payments to producers for reporting sick animals, indemnity payments for destroyed livestock, and policies to identify diseased animals may all potentially increase the stock of disease. Perverse incentives may be mitigated in some cases through changes in payments for submitting sick animals for testing and other activities that would aid in disease detection. The socially optimal level of payments, however, will depend on the level of monitoring and other variables. Our conceptual analysis highlights the significance of designing the right combination of regulatory and incentive-based policies.

2 Model of producer behavior with endogenous risk of detection

A representative livestock producer maximizes the expected economic benefits from livestock sales and from government payments before and after the disease is detected by the government. Upon detection, the presence of the disease becomes public knowledge, and prices fall due to lowered demand domestically and/or internationally. The model includes three state variables and two control variables. The first state variable is c_t, the stock of animals (e.g. livestock) at time t.

The second state variable is q_t, the stock of the disease in the population. We model the disease level directly rather than the number of infected animals, as in Bicknell et al. (1999). This formulation is general enough to include diseases that continue to spread infection after the death of the host animals.

The probability that the presence of the disease in the population is detected by the government is treated in the form of a third state variable, endogenizing the risk faced by producers at each point in time. We model this endogenous probability using a survivor function, following previous work on risks from environmental catastrophe (e.g. Clarke and Reed 1994; Gjerde et al. 1999).

In each time period, the livestock producer faces an instantaneous probability of disease detection denoted $\dot{\lambda}(t)$ conditional on disease detection having not yet occurred. We specify the timing of disease detection as a random variable, and, for tractability assume this variable has a Poisson distribution[1] . Conditional on disease detection having not yet occurred, the probability of the government detecting the disease in any interval d t is $\dot{\lambda}(t)\mathrm{d}t$ where $\lambda(t) = \int_0^t \dot{\lambda}(t)\mathrm{d}s$. If T is a stochastic variable that represents the time of disease detection in the entire population of infected animals, the cumulative probability density function for disease detection is $F(t) = \Pr(T < t)$ and $F(t) = 1 - \mathrm{e}^{-\lambda(t)}$ given the Poisson specification. The survivor function is $S(t) = Pr(T > t) = 1 - F(t)$ which equals the probability that the livestock producer continues to market animals without the government detecting the disease up to time period t. Under the Poisson specification, the survivor function is $S(t) = \mathrm{e}^{-\lambda(t)}$ so the probability of survival up to time t without detection and subsequent detection at time t is $\dot{\lambda}(t)\mathrm{e}^{-\lambda(t)}$.

In our model, the livestock producer affects c_t as well as the disease stock and the detection risk by choosing two variables at each point in time: h_t, the livestock level harvested (and marketed), and d_t, the level of "reporting." Reporting activities are any actions by producers that increase the likelihood of the government detecting the disease. This embodies the idea that producers have private information regarding the likelihood that their animals are infected, which is not available to the government regulator. Thus, producers have the choice of taking private actions, such as submitting sick animals for governmental testing, which increase the chances of disease detection. Such activities could entail private costs to the producer and may not be undertaken without governmental incentives. Conversely, producers could take actions such as concealing sick animals from government monitors that would reduce the chances of disease detection.

Given this framework, the producer's problem is to maximize the present discounted value of an infinite stream of livestock harvests, net of carrying costs, plus net benefits from reporting (e.g. government incentive payments minus private costs). The producer's problem

[1] Often used to represent counts of events across time, a Poisson distribution assumes that the probability of observing an event is approximately proportional to the size of a time interval; that there is virtually no probability of two events occurring within the same interval; that the process determining the probability does not change over time; and that the probabilities are independent across intervals. While these assumptions will affect the exact results, our model serves to illustrate potential producer behavior given endogenous risks of detection.

is thus to choose levels of h and d in each period t to maximize:

$$J = \int_0^\infty \left\{ \pi_0 h(t) - c(t)f(t) + z(t)d(t) + \dot{\lambda} v(t) \right\} e^{-\lambda(t)} e^{-rt} \partial t \tag{1}$$

where π_0 is the unit price of livestock prior to disease detection; $h(t)$ the amount of livestock harvested (and sold); $f(t)$ the carrying (feeding) cost for a unit of livestock; $z(t)$ the net reward faced by a producer for each unit $d(t)$ of reporting activity; $v(t)$ denotes the value function in the post-detection scenario; and r is the instantaneous discount rate. The producer's choice in (1) is subject to the state equations for livestock, disease, and risk evolution, (2), (3), and (4) below.

Dropping the time notation henceforth for simplicity, the livestock population increases as a function of the existing stock c, times a fixed growth rate ρ, and declines with harvest level h, and disease stock q, where u denotes the contribution of the disease stock to animal mortality:

$$\dot{c} = \rho c - h - uq \tag{2}$$

The disease stock q, increases with c and an exogenous component θ:

$$\dot{q} = cq - \theta \tag{3}$$

The disease stock evolves in proportion to its existing level times the livestock level net of spontaneous introduction or remission of the disease. Negative (positive) θ implies disease increases (decay) over time due to exogenous effects. The multiplicative term reflects a contagious disease directly transmitted across living animals so that the greater the disease stock and the animal population, the greater the infection rate in each time period. The formulation also applies to cases where transmission occurs through other pathways, such as feed contaminated with tissues from infected animals.

The instantaneous probability of disease detection is modeled as an additive function of the disease stock, the level of reporting, and a function that depends on the amount of government monitoring activity m:

$$\dot{\lambda} = a_0 q + a_1 d - e^{-m} \tag{4}$$

The likelihood of disease detection depends positively on the disease stock, which affects the detection probability given some base level of surveillance activity.[2] Higher levels of governmental monitoring m also increase the detection probability. Producers also influence the likelihood of detection through reporting actions d based on private information or behavior. This formulation highlights the role of private participation in disease control. For simplicity, the marginal impacts of d or m on the detection probability are assumed independent of q. This may be realistic if producers or the monitoring agency can target testing or reporting in a manner that does not depend on the overall disease stock.

For the base case, we specify a simple post-detection scenario in which the livestock price declines when the disease is detected, but the government is able to eradicate the disease completely and prevent its future introduction[3]. In a more realistic scenario, the exogenous risks of disease evolution would remain positive and the livestock price would recover over time. The results from our base-case formulation can be generalized in a straightforward fashion to incorporate this added realism, as discussed in Sect. 3.2.

In the base-case post-detection scenario, livestock grows as:

$$\dot{c} = \rho c - h \tag{5}$$

depending only on the growth rate and harvest level, with no death from the disease. The producer realizes returns from livestock sales at a reduced price $\pi_1 < \pi_0$. The producer's objective is then to choose harvest levels to maximize the infinite stream of net benefits from marketing livestock starting at detection time T:

$$v(T) = \underset{h}{Max} \int_T^\infty (\pi_1 h - cf) e^{-rt} \partial t \tag{6}$$

subject to (5).

Restricting attention to the steady-state livestock level ($c = 0$), producers receive an infinite stream of net benefits $c(\pi_1 \rho - f)$, and the value function can be rewritten as:

$$v(T) = c \frac{\pi_1 \rho - f}{r} \tag{7}$$

While we focus on behavior in the steady state, there is no guarantee that this equilibrium exists or will actually be reached. As discussed by Clarke and Reed (1994), we assume that the steady-state solution is informative about the direction in which the system is headed. This will be true if the system is converging towards the steady-state behavior, even if equilibrium is never attained.

Substituting Eq. 6 into Eq. 1 and using the result in (7), the producer's optimization problem can be solved using Pontryagin's maximum principle. The current value Hamiltonian is:

$$H = \left\{ \pi_0 h - cf + zd + \dot{\lambda} v \right\} e^{-\lambda} + l_1 \dot{c} + l_2 \dot{q} + l_3 \dot{\lambda} \tag{8}$$

where l_1, l_2, and l_3 are, respectively, the shadow prices with respect to c, q, and λ. Due to the Hamiltonian being

[2] Equation 4 only applies to positive and non-zero levels of disease.

[3] For simplicity, we assume that the animal numbers culled to achieve disease eradication are minor and do not to affect the livestock owner's incentives

linear in the control variables, we look for optimality conditions around the steady state. Substituting (2), (3), and (4), the first order necessary condition for an optimum with respect to the harvest level h is:

$$\frac{\partial H}{\partial h} = \pi_0 e^{-\lambda} - l_1 = 0 \tag{9}$$

The first order condition with respect to reporting is:

$$\frac{\partial H}{\partial d} = z e^{-\lambda} + a_1 v e^{-\lambda} + l_3 a_1 = 0 \tag{10}$$

Further, the rate of change of shadow prices is given by:

$$\begin{aligned}\dot{l}_1 &= -\frac{\partial H}{\partial c} + r l_1 = \frac{\partial l_1}{\partial t} \\ &= -\left\{ -f e^{-\lambda} + \dot{\lambda}\left(\frac{\pi_1 \rho - f}{r}\right) e^{-\lambda} + l_1 \rho + l_2 q \right\} + r l_1\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{l}_2 &= -\frac{\partial H}{\partial q} + r l_2 = \frac{\partial l_2}{\partial t} \\ &= -\left\{ v(t) a_0 e^{-\lambda} - l_1 u + l_2 c(t) + l_3 a_0 \right\} + r l_2\end{aligned} \tag{12}$$

$$\dot{l}_3 = -\frac{\partial H}{\partial \lambda} + r l_3 = \frac{\partial l_3}{\partial t} = \left\{ \pi_0 h - f c + d + \dot{\lambda} v \right\} e^{-\lambda} + r l_3 \tag{13}$$

These necessary conditions will also be sufficient for maximization of the Hamiltonian if it is jointly concave in both the state and control variables (Mangasarian's theorem).[4] In this analysis, we assume the conditions for sufficiency are satisfied (see Kamien and Schwartz 1981 for further details).

The steady state requires $\dot{l}_1 = 0$, $\dot{l}_2 = 0$,and $\dot{l}_3 = 0$,. Transforming $l e^{\lambda}$ into present value shadow prices μ, we obtain:

$$f - \dot{\lambda}\left(\frac{\pi_1 \rho - f}{r}\right) - \mu_1 \rho - \mu_2 q + r \mu_1 = 0 \tag{14}$$

$$-v a_0 + \mu_1 u - \mu_2 c - \mu_3 a_0 + r \mu_2 = 0 \tag{15}$$

$$\pi_0 h - f c + z d + \dot{\lambda} v + r \mu_3 = 0 \tag{16}$$

Further, $\frac{\partial c}{\partial t} = 0, \frac{\partial q}{\partial t} = 0$,and $\frac{\partial \lambda}{\partial t} = 0$ imply:

$$h = \rho c - u q \tag{17}$$

$$q = \frac{\theta}{c} \tag{18}$$

The steady-state harvest level equals the growth in livestock net of death from disease. In the steady-state, the change in disease stock from contagion equals cq which is completely offset by exogenous decay θ.[5] The cumulative probability distribution given by λ remains constant as the impact on the detection probability of reporting behavior, monitoring, and disease levels are balanced as follows:

$$d = \frac{e^{-m}}{a_1} - \frac{a_0}{a_1} q \tag{19}$$

Equations 14–19 and first order conditions given by (9) and (10) comprise eight equations in these eight unknowns: c,q,d,h,μ_1, μ_2, μ_3, and λ.

3 Results

This section examines how the steady-state levels of the state variables change with model parameters.[6] We emphasize the impacts of policy parameters on the livestock level, which is inversely proportional to the steady-state disease stock as shown in (18). We first discuss comparative statics and then illustrate the system's dynamics through numerical simulations.

3.1 Comparative static analysis

Using (7) and (14)–(19), an implicit function for the steady-state level of μ_3, the rate of change of the shadow price of detection probability λ, is derived in terms of the model parameters:

$$\begin{aligned}G &= \dot{\mu}_3 \\ &= c^2(\pi_0 \rho - f - a_1(\pi_1 \rho - f)) - c\left(\frac{z}{a_1}\right)(r - e^{-m}) \\ &\quad - \left(\pi_0 u \theta + \frac{z a_0 \theta}{a_1}\right) \\ &= 0\end{aligned} \tag{20}$$

This equation is quadratic in the steady-state livestock level. We illustrate the shape of this function with hypothetical values for the model parameters. Figure 1 shows an example of $\dot{\mu}_3$ varying with the livestock level. Because the parameter values are purely hypothetical, the livestock numbers are a general indicator of the livestock level, rather than a direct measure of animal numbers.

[4] This would require that the 5 × 5 Hessian matrix comprising the second order partial derivatives of the three state and two control variables is negative semi-definite. In order to establish negative semi-definiteness, it must be shown that all the principal minors have discriminants that alternate in sign, with the first one being negative.

[5] In examining the steady-state solution, we assume the existence of a steady state in which monitoring and an exogenous decay of the disease stock lead to constant λ and q.

[6] For diseases that do not experience any exogenous decay, there may not be a steady-state disease level. Because a steady-state analysis is only a comparison of relative values, however, it may still be possible to redefine variables in order to study their steady-state behavior even in the case of no exogenous disease decay.

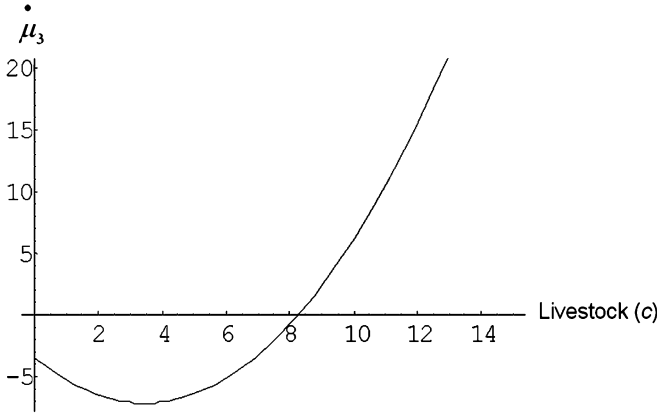

Fig. 1 Rate of change of shadow price of detection risk with respect to steady-state livestock level. *Parameters* $a_0 = 0.2$, $a_1 = 0.7$, $f = 0.4$ $m = 10$, $r = 0.05$, $t = 10$, $u = 0.6$, $z = 30$, $\pi_0 = 5$, $\pi_1 = 1$, $\rho = 0.1$, $\theta = 0.3$. Results for solutions with negative livestock are omitted

The U-shaped function in Fig.1 indicates that livestock benefits beyond a certain threshold level exceed the impact of additional animals on the detection risk from increased disease growth. Under alternative parameters—for example, if the disease growth were highly susceptible to livestock levels or if livestock mortality due to disease were high—then the rate of change of shadow price of λ could follow an inverted U-shaped curve with respect to livestock levels.

Although the shape of the function depends on the chosen parameters, Fig.1 shows how the shadow price of increased detection risk depends on the livestock level. In the U-shaped case, at high livestock levels, the rate of change of shadow price of increased detection risk is positive while at lower livestock levels, it is negative, with $\dot{\mu}_3 = 0$for a level of c around δ. When post-detection value is low compared to pre-detection benefits, λ could be a "bad" from a producer's perspective. However, if benefits to producers in the post-detection scenario were to exceed those in the pre-detection scenario, λ could be a "good" so it would pay to increase the probability of disease detection. Post-detection benefits would be greater with higher prices of livestock (or greater market share) for some producers or indemnity payments from the government.

To examine impacts from government policies, we conduct a comparative static analysis of the steady-state livestock level with respect to key exogenous parameters in the model. Using the implicit function theorem, we obtain partials of the livestock level with respect to the different parameters, with particular emphasis on those, which the government can directly influence. Understanding the impacts of key variables on the livestock level helps in understanding the impacts on the steady-state disease stock. As indicated in (18), in the steady state, livestock and disease stock are inversely related in proportion to the exogenous disease decay parameter θ. Depending on this exogenous factor, livestock and disease must remain in a fixed proportion to maintain the steady-state level of disease detection risk. As the steady-state livestock level rises, the steady-state disease stock falls, and vice versa.

We first consider the change in the steady-state livestock with respect to the pre-detection price (π_0):

$$\frac{\partial c}{\partial \pi_0} = -\frac{\partial G/\partial \pi_0}{\partial G/\partial c} = \frac{u\theta - c^2\rho}{2c\{\pi_0\rho - f - a_1(\pi_1\rho - f)\} - \frac{z}{a_1}(r - \mathrm{e}^{-m})} \tag{21}$$

This equation reflects the tradeoff in terms of balancing risk of detection and increased mortality due to disease from a marginal increase in livestock versus the increased benefits from the marginal unit of livestock in terms of current and future harvests. The denominator (which is the same in all of the partials of c) equals the change in $\dot{\mu}_3$ with respect to the livestock level in the steady state. As such, it is the partial derivative of the instantaneous expected benefits with respect to a marginal change in steady-state livestock. The sign of this term varies depending on whether or not the benefits of an additional livestock unit in the pre-detection world exceed the costs of reducing reporting to compensate for the added detection risk from this additional livestock.

In the pre-detection scenario, the benefits from an additional unit of livestock are the added profits from greater harvest and stock growth minus the additional carrying costs f and the foregone benefits in the post-detection scenario. These effects comprise the terms $2c\{\pi_0\ \rho - f - a_1\ (\pi_1\ \rho - f)\ \}$ in the denominator. The bracketed terms are magnified by the livestock level. This dependence arises because both livestock growth and disease growth depend upon the livestock level. Livestock growth (before harvest and mortality) equals the stock times the constant growth rate ($c\rho$) in (2) and (5). The dependence of disease on the livestock arises from the biological feature of disease contagion embodied in the term cq in Eq.3. This term indicates that the chances of disease spread increase in proportion to the size of the animal population. As a result, when the steady-state livestock level is high (and disease levels are thus low), the added risk produced on the margin by an extra unit of disease is greater than when the livestock population is lower (and disease higher). As a result, the cost-benefit tradeoff in terms of added risk is relatively more favorable to increasing livestock versus reporting when the livestock level is higher. This relationship provides an essential feature of the comparative static results discussed further below.

An additional unit of livestock raises the growth rate of the disease, which in turn increases the risk of detection. Thus, reporting must be lowered as disease rises to maintain a steady-state level of risk. The term $\frac{z}{a_1}(r - \mathrm{e}^{-m})$ captures the cost of marginal livestock unit on forgone benefits from reporting. As the effectiveness (a_1) of reporting increases, reporting levels need to be reduced by less for the same reduction in risk. Thus, as a_1 increases, fewer benefits from reporting need to be foregone for each additional unit of livestock. Also, the level of monitoring augments the effect of the discount rate r as the term e^{-m} decreases with m. The effect of the monitoring level is to increase the importance of reporting benefits. When monitoring is lowered, the term $(r - e^{-m})$ may actually turn negative, reversing the impact on livestock of the exogenous variable, if the denominator were negative. This dependence on the monitoring level is discussed further below.

The net effect on the sign of the partial in (21) depends on the sum of the two denominator terms, as well as the sign of the numerator. As long as the benefit of an added unit of livestock exceeds the opportunity cost in terms of foregone reporting, the denominator will be positive. For a given value of all exogenous variables, there will be a threshold livestock level above, which the sign of the partial will change. The switch in the signs of Eq. 21 depending on the livestock level is depicted in Fig.2. For values that produce a negative numerator, there is a level of stock above which the partial is negative and, below which, it is positive. This change in sign depending on the livestock level is a feature of all the partials for the livestock in the steady state.

The denominator captures the benefits of additional livestock versus reporting. The numerator reflects the tradeoff between additional livestock and a higher level of risk in the steady state. While higher prices in the pre-detection period increase current benefits from livestock, additional animals increase risk by transmitting disease, increasing the chances of detection and transition into the post-detection scenario with lower prices. This tradeoff is captured in the numerator, which is the marginal change in $\dot{\mu}_3$ resulting from the marginal change in the exogenous variable, the pre-detection price.

The numerator is the partial derivative of the instantaneous expected benefits resulting from the change in risk with respect to a marginal change in the exogenous variable. The first term in the numerator of (21) shows that the cost of the greater risk resulting from the higher livestock level in response to an increase in prices is mitigated by the death rate of livestock from the disease and the exogenous disease decay. The increment in risk is augmented by the livestock growth rate as given in the second term ($c^2 \rho$). If the parameters and steady-state levels are such that this second numerator term outweighs the first, then marginal livestock adds so much risk that the optimal response to higher livestock prices is to reduce livestock to maintain marketing benefits in the pre-detection state.

Carrying costs f will potentially be affected by government policies such as regulations on certain types of feed, which increase the unit costs of maintaining livestock. The change in steady-state livestock with respect to f is:

$$\frac{\partial c}{\partial f} = \frac{c^2(1 - a_1)}{2c\{\pi_0\rho - f - a_1(\pi_1\rho - f)\} - \frac{z}{a_1}(r - e^{-m})} \tag{22}$$

In this partial, the change in sign depends on the level of stock, but in the opposite direction than (21), with a positive partial at high livestock levels and a negative partial at low livestock levels (Fig.3). The reverse direction makes sense because the impact of f is to decrease the value of livestock while higher prices increase the value of this stock. The instantaneous benefits from a change in risk resulting from higher f decrease in a_1 and the discount rate. This is because the value of pre-detection livestock (which is now costlier to maintain) increases in these variables relative to the values of reporting and post-detection profits.

The partial with respect to the post-detection price-level is:

$$\frac{\partial c}{\partial \pi_1} = \frac{c^2 a_1 \rho}{2c\{\pi_0\rho - f - a_1(\pi_1\rho - f)\} - \frac{z}{a_1}(r - e^{-m})} \tag{23}$$

As long as the value of additional livestock sales exceeds the foregone benefits from reporting (e.g. the denominator is positive), an increase in post-detection prices will increase livestock. In this case, higher post-detection profits increase the benefits from adding

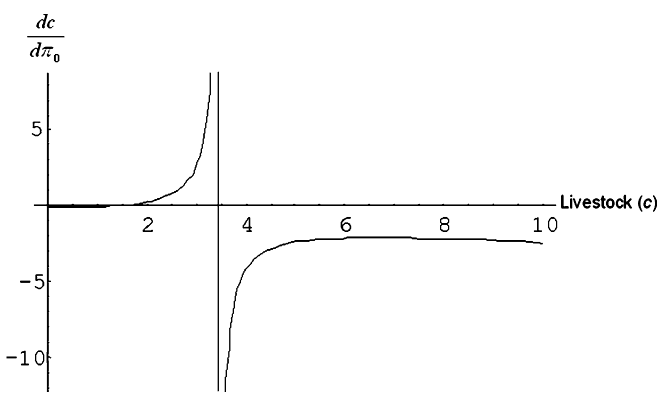

Fig. 2 Change in steady-state livestock level with respect to current prices. *Parameters* $a_0 = 0.2$, $a_1 = 0.7$, $f = 0.4$, $m = 10$, $r = 0.05$, $t = 10$, $u = 0.6$, $z = 30$, $\pi_0 = 5$, $\pi_1 = 1$, $\rho = 0.1$, $\theta = 0.3$. Solutions with negative livestock are omitted

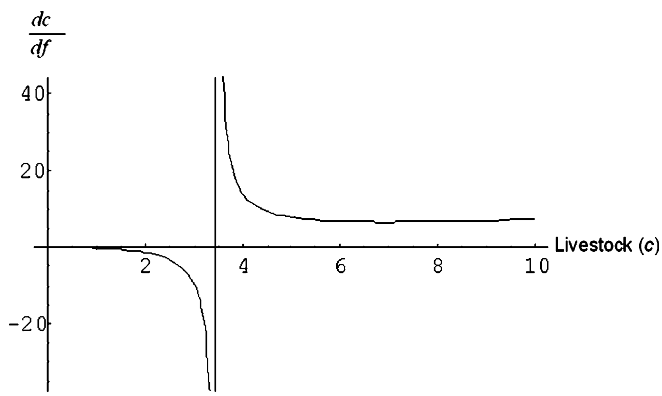

Fig. 3 Change in steady-state livestock level with respect to carrying costs. *Parameters* $a_0 = 0.2$, $a_1 = 0.7$, $f = 0.4$, $m = 10$, $r = 0.05$, $t = 10$, $u = 0.6$, $z = 30$, $\pi_0 = 5$, $\pi_1 = 1$, $\rho = 0.1$, $\theta = 0.3$. Solutions with negative livestock are omitted

detection risk through increased livestock. The benefit of adding risk through more livestock will be higher for higher levels of a_1 because this decreases the foregone benefits from reporting from higher livestock. Post-detection profits generate greater instantaneous benefits from a change in risk when the growth rate is higher and the discount rate is lower, as these raise the post-detection livestock and the value of future livestock harvests, respectively.

We now consider the effect of maintaining the livestock with respect to reporting benefits (z), which could include government payments:

$$\frac{\partial c}{\partial z} = \frac{\frac{c}{a_1}(r - \mathrm{e}^{-m}) + \frac{a_0}{a_1}\theta}{2c\{\pi_0\rho - f - a_1(\pi_1\rho - f)\} - \frac{z}{a_1}(r - \mathrm{e}^{-m})} \qquad (24)$$

For the particular values of the exogenous variables, higher reporting benefits imply the denominator is negative at low livestock levels (Fig.4). Thus, when the livestock level is low (and benefits for reporting are sufficiently high), the denominator is negative and an increase in reporting benefits would further reduce the relative benefits of livestock versus reporting, further lowering the steady-state livestock level. Beyond a threshold livestock level (about $c = 3.5$ in the example), however, the partial becomes positive and greater reporting rewards increase the amount of livestock in the steady state.

The numerator indicates that the instantaneous benefits from a change in risk in response to z increases in a_1, as reporting benefits can be obtained for less additional detection risk, and in c, a_0, and θ, as less livestock must be foregone to offset the additional detection risk. The monitoring level enters both the numerator and denominator to adjust the discount rate for the change in the detection risk. Both the magnitude and direction of the impact of reporting benefits on livestock could depend on the monitoring level as this can potentially switch the sign of both the numerator and the denominator if $\mathrm{e}^{-m} > r$. For this to happen, monitoring must

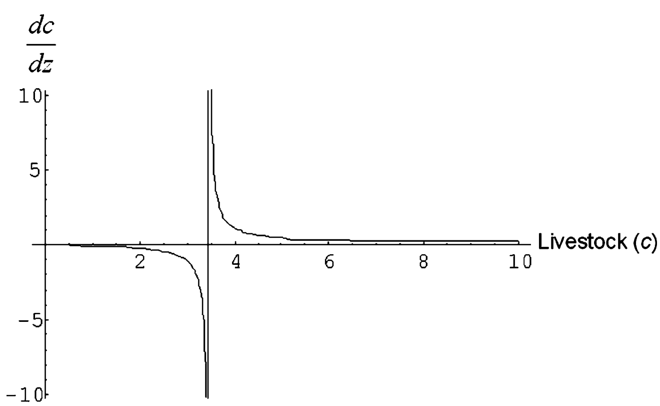

Fig. 4 Change in steady-state livestock level with respect to reporting rewards. *Parameters* $a_0=0.2$, $a_1=0.7$, $f=0.4$, $m=10$, $r=0.05$, $t=10$, $u=0.6$, $z=30$, $\pi_0=5$, $\pi_1=1$, $\rho=0.1$, $\theta=0.3$. Solutions with negative livestock are omitted

fall below some critical level m^*. Earlier we saw that the response to increasing reporting benefits was to lower livestock at high levels of reporting benefits. However, the incentives are reversed for monitoring below m^*. This highlights the role of designing the optimal mix of public policies to reach the desired objectives.

Equation 25 indicates the relationship between livestock and monitoring:

$$\frac{\partial c}{\partial m} = \frac{\frac{cz}{a_1}\mathrm{e}^{-m}}{2c\{\pi_0\rho - f - a_1(\pi_1\rho - f)\} - \frac{z}{a_1}(r - \mathrm{e}^{-m})} \qquad (25)$$

An interesting feature of this equation is that the impact of monitoring on steady-state livestock will vary based on the monitoring level. Under high monitoring, chances of detection are higher, thus making increases in livestock more costly in terms of foregone current reporting benefits. This implies that higher monitoring will lower steady-state livestock. This incentive is augmented at high levels of reporting rewards, adjusted for the contribution of reporting to risk in the term z/a_1. The reporting benefits will be less important at higher livestock levels because, simplifying further, c drops out except for the last term in the denominator which becomes $z/ca_1\ (r - \mathrm{e}^{-m})$.

Under low monitoring, the sign of the denominator could switch from negative to positive. When reporting benefits are relatively high, greater monitoring can switch the tradeoff towards increasing livestock and away from reporting. At low enough levels of monitoring risk, producers are willing to raise livestock despite high reporting rewards. The monitoring level in this equation serves to augment the effects of the market discount rate by increasing detection risks.

The comparative static relationships described above illustrate the risk management tradeoffs that govern a producer's responses to different possible policies for livestock disease management. The results suggest potentially perverse policy outcomes given the public health implications of livestock diseases. Policies that increase benefits from livestock (such as subsidies to producers), that increase costs of carrying livestock (such as regulations on feed), that reduce post-detection livestock losses (through improved tracking and surveillance), that offer payments for reporting, or that increase monitoring can each lead to either increases or decreases in the livestock (decreasing and increasing the disease level, respectively). Both the magnitude and direction on the steady-state disease stock will depend on the value of all of the exogenous parameters, as well as the steady-state level of livestock. For example, Eq. 21 shows that for the case of $r > \mathrm{e}^{-m}$ policies that increase livestock prices will increase livestock and reduce steady-state disease only when the livestock level (and growth rate) is relatively low compared to the lethality and decay rate of the disease. This effect could be reversed at low livestock levels.

The comparative static results underscore the importance of selecting an efficient mix of incentive-based policies and monitoring. Under low monitoring levels, higher payments for rewarding will have the opposite effects compared to a situation where monitoring is high. Policies that create incentives to increase the disease stock might increase the livestock sector's profits, but may not necessarily increase the overall public good. Understanding the risk calculus of producers is thus essential for making policy adjustments and developing an efficient portfolio of government interventions.

3.2 Alternative scenarios for post-detection values

Our analysis illustrates certain elements of optimizing behavior under simple assumptions about the nature of disease spread and livestock dynamics. Several additional complexities are worth considering, such as a high sensitivity of import demand to an outbreak of the disease. In this case, the fall in prices after detection may be related to the extent of the disease in the environment. This is reflected in the following post-detection value function:

$$v(T,q,c,r,\pi(q)) = \int_t^{\infty} (\pi_1(1-lq)h - cf)\mathrm{e}^{-rt}\partial t \tag{26}$$

where l is the parameter measuring the impact of the disease stock on prices upon detection. In this case, producers will have additional incentives to reduce the disease. Similarly, if indemnities i are provided in case of disease detection and indemnities are based upon the level of q, then the post-detection value function becomes:

$$v(T,q,c,r,\pi,\rho) = \int_t^{\infty} (\pi_1 h - cf)\mathrm{e}^{-rt}\partial t + iq\mathrm{e}^{-rt} \tag{27}$$

Given (27), producers would face greater incentives to report but could face perverse incentives to raise the level of q to increase detection risk and thus obtain indemnities. Under a combination of (26) and (27), indemnities and price losses would, respectively, decrease and increase the costs (incentives to avoid) disease detection.

Another potentially relevant scenario is one in which the disease cannot be eliminated completely and recurs after detection. For simplicity, consider a case where the second detection leads to a total shutdown of the industry. The value function after the second detection is:

$$v(T,q,c,r,\pi,\rho) = 0 \tag{28}$$

For the period between the first and the second detections, the current value Hamiltonian is:

$$H = \{\pi_1 h - cf + zd\}\mathrm{e}^{-\lambda t} + l_1 c^{\bullet} + l_2 q^{\bullet} + l_3 \lambda^{\bullet} \tag{29}$$

The owner's objective is to maximize the sum of the discounted value of livestock marketing returns and reporting benefits net of the carrying costs. In contrast to Eq. 8, λ in this equation serves only as an additional discounting term because the producer receives no benefits in the post-detection scenario. Falling prices after the first detection lower the optimal steady-state livestock level as shown by Eq. 23.[7] The steady-state livestock level solves the equation below:

$$c^2 - rc - \frac{\pi_1 u\theta + z\theta\frac{a_0}{a_1}}{f + \pi_1(r-\rho)} = 0 \tag{30}$$

Equation 30 is a quadratic form whose roots are given by:

$$c = \frac{r}{2} \pm \sqrt{r^2 + 4\frac{\pi_1 u\theta + z\theta\frac{a_0}{a_1}}{f + \pi_1(r-\rho)}} \tag{31}$$

The value function after the first detection is now:

$$v = \int_t^{\infty} \left\{\pi_1\rho c - cf + z\left(\frac{\mathrm{e}^{-m}}{a_1}\right) - \frac{a_0}{a_1}\frac{\theta}{c}\right\}\mathrm{e}^{-rt} \tag{32}$$

where the first term in the integral is the steady-state benefits from livestock harvest, and the third term is the benefits from steady-state reporting activities. Using these equations, the Hamiltonian is:

$$H = \{\pi_0 h(t) - c(t)f + zd(t) + \lambda v\}\mathrm{e}^{-\lambda t} + l_1 c^{\bullet} + l_2 q^{\bullet} + l_3 \lambda^{\bullet} \tag{33}$$

This Hamiltonian differs from (8) in that there is now a constant benefit after the first detection. The steady-state value of livestock satisfies the implicit function:

$$c^2 - rc - \frac{\pi_0 u\theta + z\theta\frac{(a_0+v)}{a_1}}{f + \pi_0(r-\rho)} = 0 \tag{34}$$

The roots of this equation are:

$$c = \frac{r}{2} \pm \sqrt{r^2 + 4\frac{\pi_0 u\theta + z\theta\frac{(a_0+v)}{a_1}}{f + \pi_0(r-\rho)}} \tag{35}$$

It is interesting to compare the steady-state values of livestock before first and second detections. Note that the steady-state value of livestock given by Eq. 31 after the first detection would be higher than the steady-state value before the first detection (35) as long as the effect

[7] The producer will increase livestock as compared to the steady-state if the post-detection prices actually rise or if the indemnities paid by the government ex-post are sufficiently high. To model these cases, the value of livestock after the first detection would have to be broken into two parts. The first part would equal the stream of benefits from livestock until the livestock reaches its steady-state value and the second part would equal the stream of benefits at the steady-state value.

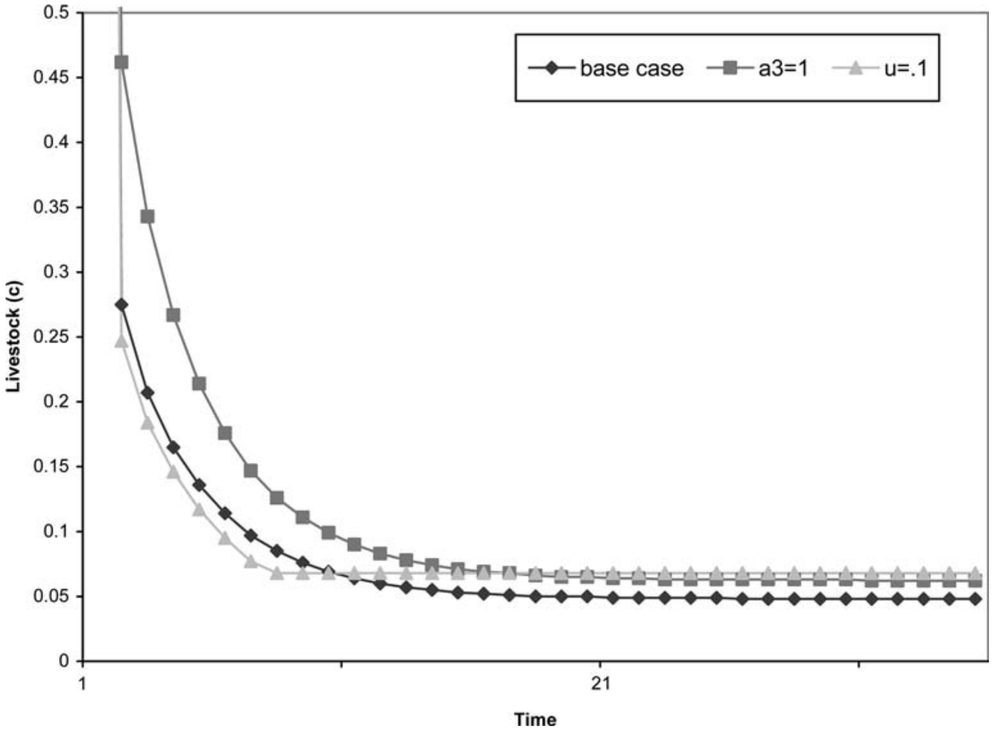

Fig. 5 Evolution of livestock under alternative parameter values. *Parameters* $m=100$, $r=0.05$, $\rho=0.05$, $a_0=0.2$, $a_1=0.01$, $u=0.01$, $z=0.005$, $\theta=0.003$, $f=0.681$, $\pi_0=2.5$, $\pi_1=0.95$, $a_3=1.3$, $q_0=0.005, c_0=3, \lambda_0=0.01$

of lower profits π_0 in Eq. 35 dominates the extra term (V) within the roots, which could be negative. Intuitively, as the number of detections increase and the value from post-detection livestock falls, it pays to incorporate the impact of current actions on future losses in advance. Numerical derivation of the steady state level of livestock would confirms the intuition that multiple detection scenarios imply a lower optimal livestock. This suggests that expectations about continuing government efforts—and how they will affect future livestock profits—will be important in shaping livestock owners' risk mitigation decisions.

So far we have focused on comparisons of state variables. However, it is also important to examine the dynamics involved with the non-linear nature of disease evolution. We explore these issues in Sect. 3.3.

3.3 Numerical analysis of the dynamics

Given the non-linear nature of the state equations, examining the time path of the key policy variables such as livestock, disease, and reporting may provide some insights into potential policy effects. We briefly explore the role of some key parameters on the system dynamics using numerical simulations. Figure 5 shows the time path of the livestock level under various situations. The base case reflects a set of hypothetical parameter values that were selected for producing steady-state livestock and disease levels, as shown in Figs. 5 and 6, respectively. Note that the base case assumes that there are negative rewards from reporting activites.

While the livestock level falls to a steady-state level in the base case, when the growth rate of disease—affected by the stocks of both livestock and disease—is lowered exogenously, the livestock level falls initially but rises later on. A parameter termed a_3 changes the degree of disease transmission mechanism in Eq. 3.[8]. This could reflect regulatory measures, such as restrictions on livestock feed that could reduce disease spread. A lower impact of the stock of disease and livestock to the growth rate of disease would allow for a larger livestock level. The livestock level falls initially along an optimal path up to a certain point and then increases beyond it. The optimal livestock level rises at later stages after the exogenous rate of decay of disease has a higher (negative) impact on the growth rate of disease compared to the lower (but positive) impact from the combined effect of increased livestock but decreased disease. Livestock is also lower when the lethality of the disease given by u is higher. The livestock level, however, rises above the base case during later stages even though the death rate is higher. This is because the high reduction in livestock in the early stages significantly lowers the growth rate of disease in the later stages, thus allowing for a higher livestock level. This reveals the complex nature of disease dynamics that can arise based upon the initial values of key parameters.

In the base case, disease increases even as the livestock level falls (Fig. 6). Disease levels also increase the rate of disease growth (parameter a_3) is lower. This is because a lower level of disease growth encourages a higher maintenance of livestock level. Disease falls with higher disease induced livestock mortality (u) as livestock levels are reduced.

[8] Equation 3 is redefined as $\dot{q} = a_3(cq) - \theta$..

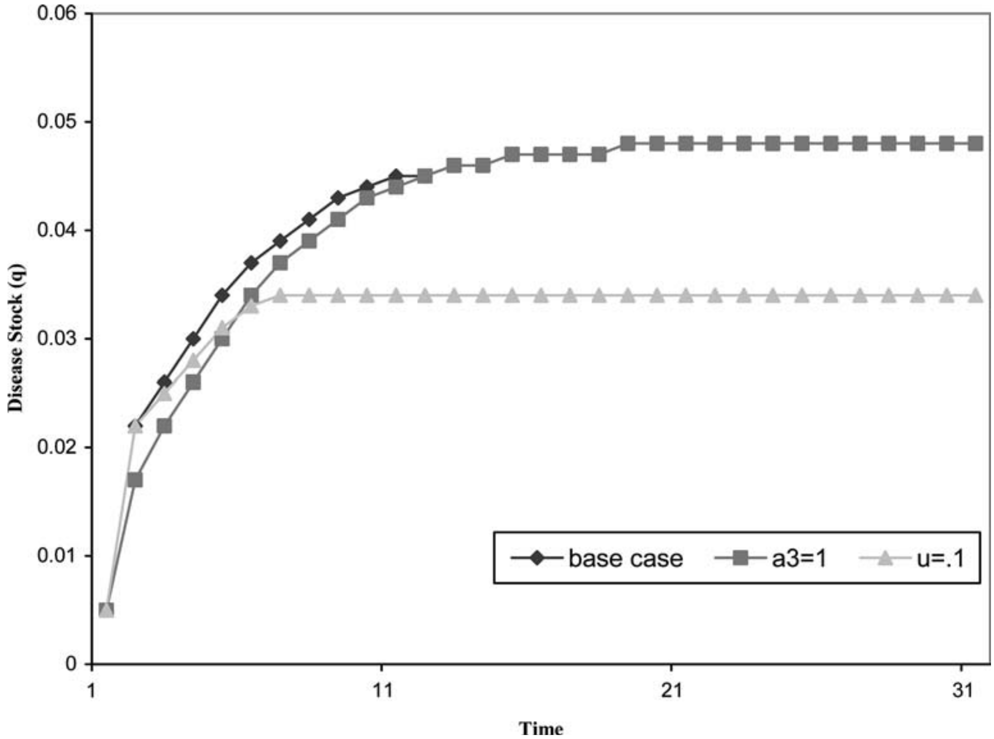

Fig. 6 Evolution of disease under alternative parameter values. *Parameters* $m = 100$, $r = 0.05$, $\rho = 0.05$, $a_0 = 0.2$, $a_1 = 0.01$, $u = 0.01, z = 0.005$, $\theta = 0.003$, $f = 0.681$, $\pi_0 = 2.5$, $\pi_1 = 0.95$, $a_3 = 1.3$, $q_0 = 0.005$, $c_0 = 3$, $\lambda_0 = 0.01$

Figure 7 shows the impact on reporting behavior under similar scenarios. There is no reporting in the base case. When a positive reward is provided for reporting activities, reporting becomes positive. As would be expected, reporting is highest when postdetection decline in the price of livestock is low (p1 = 1.95). (Nextly, for a given level of reporting rewards and death rate, reporting falls when the rate of disease spread is lower. Higher rate of disease transmission encourages a shift towards the postdetection environment, which is disease free.

This examination of the dynamic aspects of the model reveals that the time path of disease may follow counter-intuitive patterns. These responses would be difficult to explain without understanding the underlying patterns of private optimizing behavior.

4 Conclusion

This analysis explored the behavioral aspects of livestock disease management from the livestock owner's perspective. We developed a stochastic, dynamic model of livestock levels and disease for a representative producer who can take private actions to increase the gov-

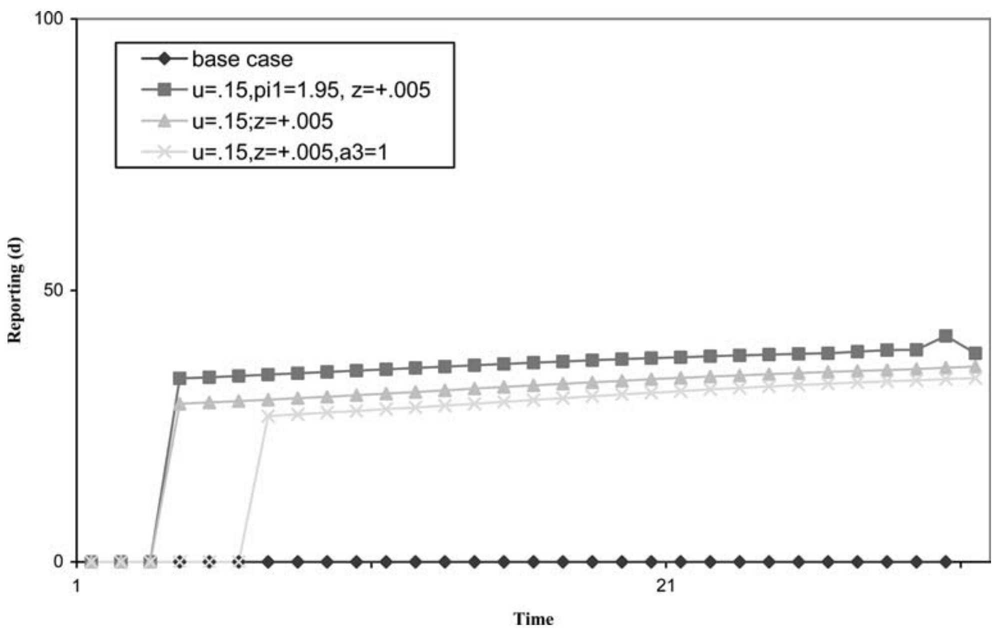

Fig. 7 Evolution of reporting under alternative parameter values. *Parameters* $m = 100$, $r = 0.05$, $\rho = 0.05$, $a_0 = 0.2$, $a_1 = 0.01$, $u = 0.01$, $z = 0.005$, $\theta = 0.003$, $f = 0.681$, $\pi_0 = 2.5$, $\pi_1 = 0.95$, $a_3 = 1.3$, $q_0 = 0.005$, $c_0 = 3$, $\lambda_0 = 0.01$

ernment's chances of disease detection. In this model, the producer maximizes expected revenues from the optimal management of livestock sales and private behavior that influences the chances of disease detection.

Several insights emerge from the comparative statics and the numerical dynamic analysis. The comparative statics indicate that it is critical for the regulator to use the efficient mix of available options, lest they should lead to perverse incentives. A dynamic analysis based on hypothetical parameter values further reveals complex interactions of the biological and economic processes that can lead to counter-intuitive behavior on the part of livestock producers faced with various sources of risk.

Future research would benefit from a better understanding of the biological processes and their relationship to the potential economic and policy responses. Additional insights could potentially be gained from modeling the variation in individual producer behavior and the relationship to the livestock industry at the national level. Operations of different sizes and types could also respond differently to prices, costs, disease, and government policies. The level and nature of disease in the national herd or in different subpopulations might also affect the risk calculus of individual producers given different levels of contagion as well as market segmentation and traceability. Realistic estimates for key parameters would also enable comparisons of producer responses to different policies in the context of actual economic and biological scenarios.

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